

Introduction

“However, such adequate self-knowledge can be reached by man only at the summit of a long ascent. ... In the present work this was achieved by our study of insight as activity, for what we mean by a unity of empirical, intelligent, and rational consciousness, has to be gathered from our study of insight in mathematics, in classical and statistical science, in common sense and its fourfold bias, in the ambiguity of things and bodies, and in the reflective act of judgment.”¹

Recently, I worked through to a new way of understanding certain spaces (with emphasis on space rather than time) and advanced somewhat in my *heuristics of seeking explanatory understanding of heuristics of seeking explanatory understanding* leading to more adequate self-knowledge.

Taking a lead from McShane’s book², I decided to try to understand the 17th century result called Boyle’s law, $P \times V = \text{Constant}$.³ Working independently the effort took about three weeks, on and off. I needed to get to sources from the tradition. I searched through various website presentations, trying to find something that might be accessible to me. Eventually, I homed in on three main “experiments.” Historically speaking, the first two are in “reverse chronological order.”⁴ But for me, that reversal proved helpful. The first is a pedagogical presentation of a “mock experiment.” Results there are “too perfect.” But the presentation helped me get a better sense of the problem and also pushed me to brush up on some school mathematics that I eventually needed in order to get into and understand Boyle’s work. Note also that the first and third “mock experiments” both rely on various modern results.

Figure 1. Mock Experiment A

A platform is attached to the top of a plunger that fits into a rigid cylindrical tube. The plunger is designed to both create an air-seal and move easily. Putting weights (in grams) onto the platform pushes the plunger to different levels. The more weights added, the further down the plunger goes. It is also observed that by removing weights, the plunger rises back to its original height. The trapped air seems to work like a spring. Reviewing some school mathematics, volume of a circular cylinder is (height) x (area of circular cross-section). In this mock experiment, the cross-sectional area of the cylinder is approximately one square cm (formula for the area of a circle).

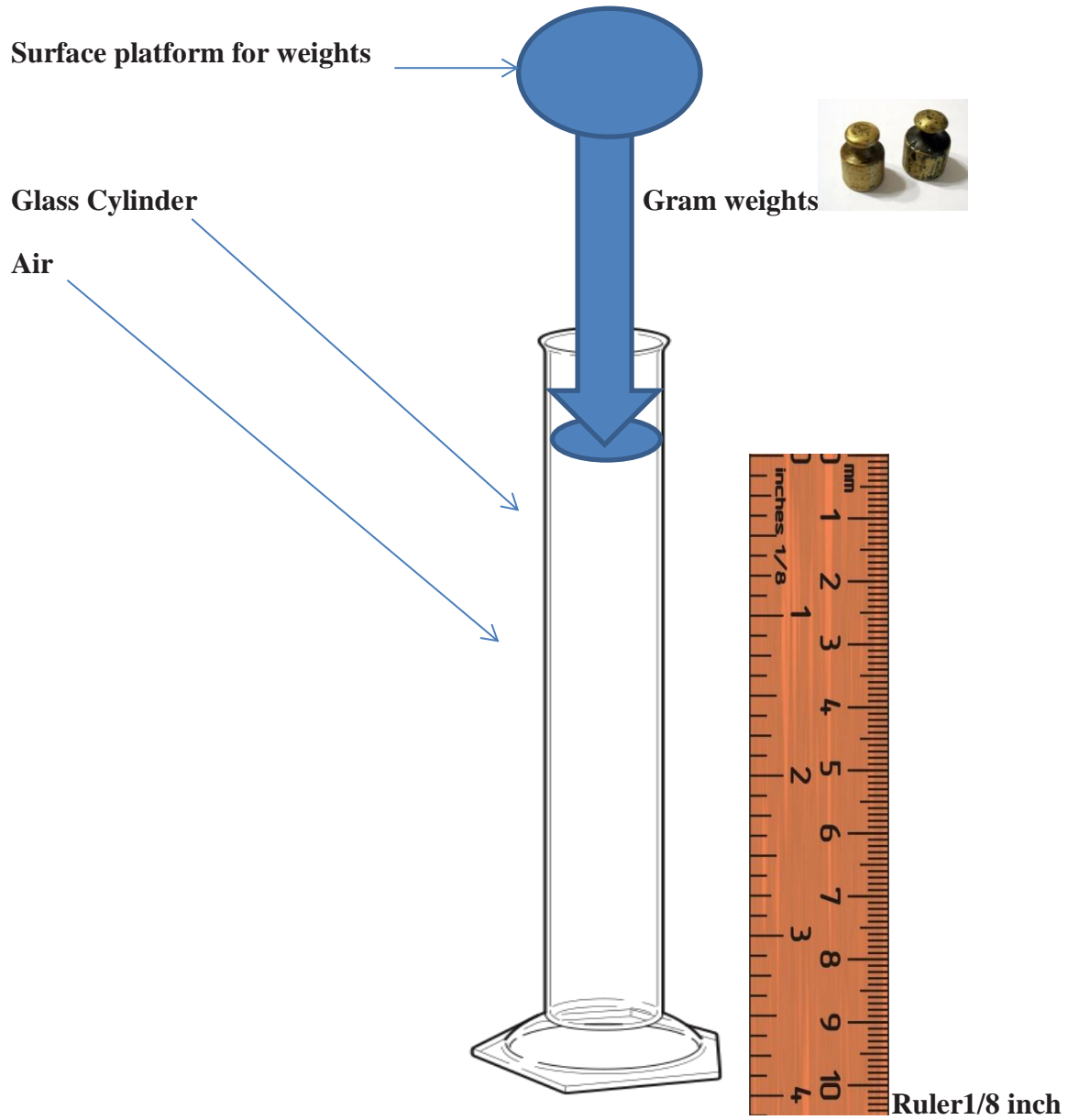
¹ Bernard Lonergan (1992) *Insight: A Study of Human Understanding*, CWL 3, University of Toronto Press, CA, p. 558.

² Philip McShane, *Wealth of Self and Wealth of Nations: Self-Axis of the Great Ascent*, (Exposition Press, NY, 1975), p. 15. Another one is in CWL 18 of Lonergan’s collected works as Appendix A Bernard Lonergan, *Phenomenology and Logic: The Boston College Lectures on Mathematical Logic and Existentialism*, CWL 18, (University of Toronto Press, 2002). pp. 322-323.

³ Philip McShane, (1975), *op. cit.*, p. 24.

⁴ The first experiment is one I devised myself as a way of exercising my imagination before finding the other two experiments. So, for me, the order of the experiments in terms of chronological understanding is correct.

And so, to measure volume of trapped air we only need to measure height of the plunger. A ruler is lined up alongside the cylinder.



I found that having results presented in a table was helpful.

Weight on plunger (grams) for “pressure” P	Height of air under plunger (cm) for volume V
1	8
2	4
3	8/3
4	2
5	8/5
6	8/6
7	8/7
8	1

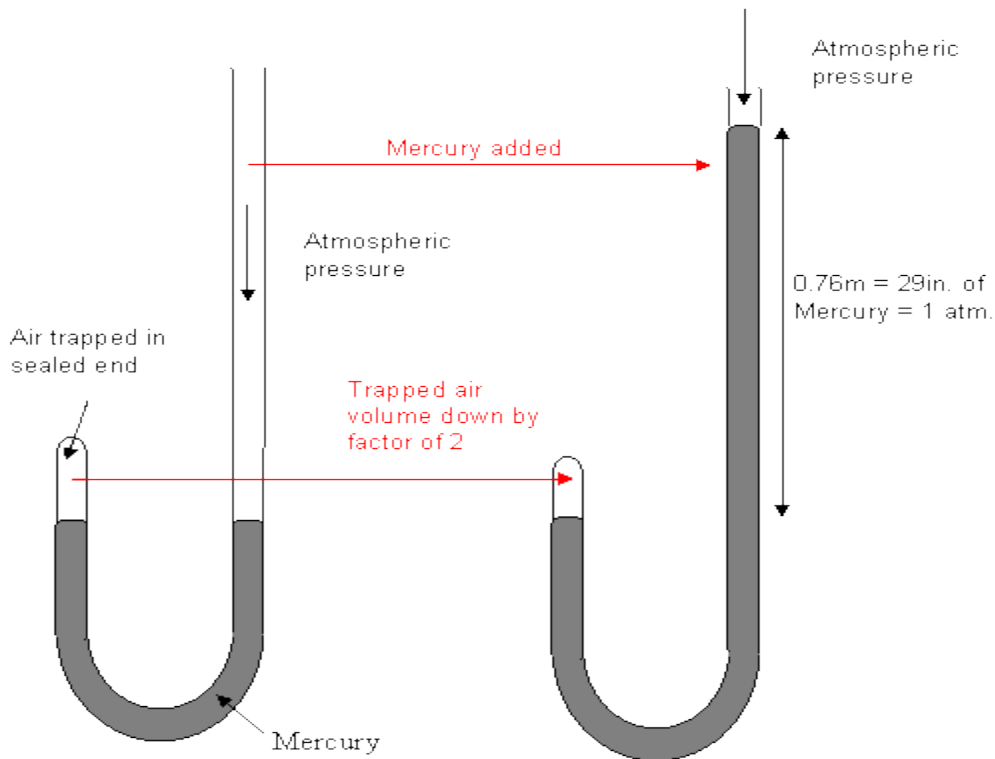
Table 1. Results from mock experiment A.

I notice that the larger the weight the smaller the volume. I have another insight. In each case, $P \times V = 8$. To express the idea that smaller volumes results in larger pressures, I also find it helpful to write this as $V = 8/P$. (Divide both sides by P.)

*Boyle’s J-tube Experiment*⁵

Boyle’s experiment involved a “J-tube,” a curved glass cylinder closed at one end.

⁵ Picture retrieved from online images June 6, 2020.



Boyle found that when more mercury was poured into the tube, increasing pressure on the trapped air, the air volume halved if the *total* pressure, including that from the atmosphere, was doubled.

Figure 2. Boyle's J-tube experiment.

By adding "volumes of liquid mercury" to an open end of a J-tube, pressure is applied to a pocket of trapped air at the closed end. See Figure 2. To begin, pour in a small amount of liquid mercury enough so that mercury settles as it happens, at the same level on both sides (as indicated in the left part of the diagram). On the hypothesis that added pressure (whatever *pressure* is) is proportional to volume of mercury added, we measure height of added liquid mercury. We also measure the volume of trapped air in the same way. It was a clever set up! But the pocket of air is much shorter than the lengths along the column of mercury. So, the height of the air pocket is measured in quarter inch steps, while liquid mercury is measured in full inch steps. Here are some results that Boyle obtained in one of his experiments⁶:

⁶ John West "The original presentation of Boyle's Law" (1999 the American Physiological Society) p. 1544.

Volume of trapped air, in quarter inches (experimental)	Pressure/volume of liquid mercury, in inches (experimental)	P(experimental) x V(experimental)
48	$29 \frac{2}{16}$	1398
46	$30 \frac{9}{16}$	1406
44	$31 \frac{15}{16}$	1405
42	$33 \frac{8}{16}$	1407
40	$35 \frac{5}{16}$	1413

Table 2. Partial results from Boyle’s experiment.

To refine his measurements, Boyle estimated (1/16th) inch increments. Looking across each row, the products of pairs of experimentally obtained P and V are not all the same (as they were in Mock Experiment A). But they are all close to being equal, to 1405, more or less. I have an insight: Allowing for “experimental error,” I understand, as a possibility, that P x V is (approximately) constant.

I have learned that Boyle used many differently sized J-tubes. With elementary tools available to him at that time, constructing tubes that worked proved to be challenging. Sometimes columns burst under pressures from large volumes of liquid mercury. But what he found was that, by allowing for experimental error, $P \times V = \text{constant}$, where the constant depended on the dimensions of the J-tube.

Figure 3. Mock Experiment B: A Demonstration of Boyle’s Law

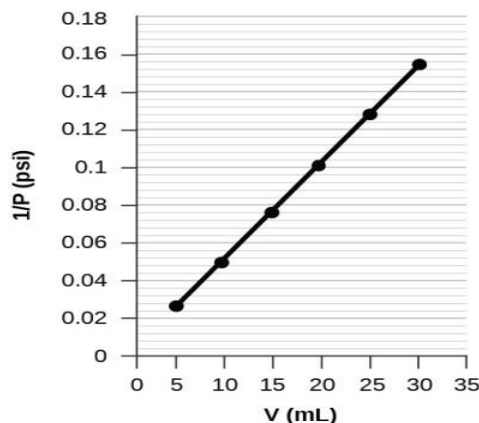
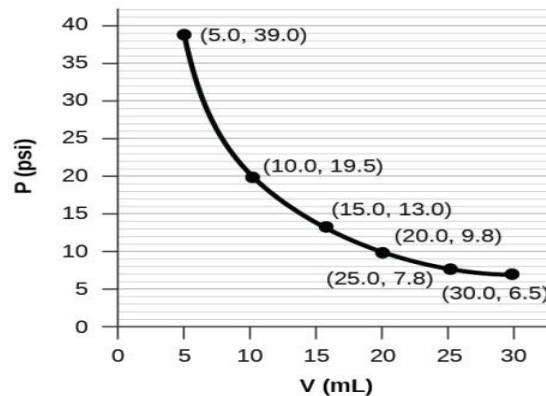
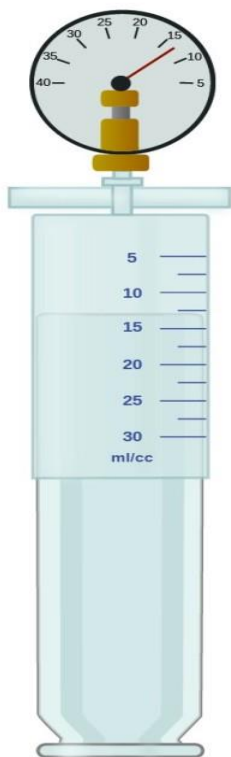
This presentation is intended to help students understand Boyle’s law. See Figure 3. It includes two (Cartesian) coordinate graphs for comparing pressures and volumes.⁷ The first compares P and V directly. The second is for comparing 1/P with volume V. A graduated cylinder⁸ is sealed with a plunger that can be pushed to various depths. A gauge is for measuring pressures of trapped air.

Each pair of measurements (one for pressure and one for volume) provides coordinates on a “coordinate graph.” By joining dots on the graph (not obtained experimentally), we get the dark curve in the picture. For large values of V, there are small values of P, and vice versa. More precisely, for each pair we get $PV = 195$. Taking a slightly different approach, if Boyle’s law is correct, then $(1/P) = (\text{constant})V$, that is, (1/P) is proportional to V. That means that in order to

⁷ Boyle was apparently aware of Descartes’ mathematical work but did not make use of coordinate graphs.

⁸ The graduated cylinder was invented by Albert Einstein in the year of 1909, in order to help with measuring the volume of a liquid.

demonstrate Boyle's law we could instead graph pairs $1/P$ and V . For if pairs $1/P$ and V are in fact proportional, then we should get a straight line, which is the second graph. Either way, we can see that the "mock experiment" "demonstrates" Boyle's law.



Discussion

My understanding of Boyle's law involves several insights. I describe and then, on a hunch, also measure pairs of lengths with rulers. I gather numerical results in tables and graphs. By a further insight (which includes ignoring "experimental error"), I discover the relationship $PV = \text{Constant}$.

I have gone beyond both description and nominal understanding. It also occurs to me that I now also have experience in understanding something about my experience of "space." No longer merely describing, I am aware of a relation between pairs of experienced and measured lengths (spaces) that, as it happens, represent pressures and volumes.

With Boyle's law in mind, I also realize that an "explanatory heuristics of heuristics" is going to be something challenging. It will need to include and be "verifiable" in my searching for and rediscovering Boyle's law.

I also note that Mock Experiment B is out of sync with how the law was discovered. But I devised the first experiment before finding the other two experiments online. So, in terms of my developing understanding of Boyle's Law, the presentation here is chronological in terms of my growing understanding.⁹ Rather than help a student wonder about pressures and volumes, and eventually discover Boyle's law, the second presentation states Boyle's law up front and asks students to follow a "demonstration." This is just one instance of what I have observed more broadly, namely, that at this time, modern education emphasizes rote work (and other "nominal understanding"), rather than inquiry toward "explanatory understanding."

There are further insights there that I will need to unpack and that will contribute to the view that would result and that I suspect, will be important for moving forward in my heuristics of explanatory heuristics. For instance, Lonergan speaks of "correlation of correlations of correlations."¹⁰ It would be good to identify such things in my experience.

Taking them at face-value, the first and third experiments bring out the fact that Boyle's law is not an isolated result but is used, verified, and demonstrated in conjunction with other understandings and technologies. This practical use and repetitive experimentation of Boyle's law does not explain why this relationship exists between pressure and volume.¹¹ For instance, historical accounts bring out that pressure due to "weight" is better understood through Newton's laws of motion¹² which in turn, are used to engineer modern pressure gauges. To provide an explanatory account and verified generalization of what is the explanation of this relationship would involve the mathematization of the results in terms of differential equations and the derivatives that would reveal a pattern in the decreasing rate of change.¹³ Such further work would reveal a more specific refinement of the cognitional operations and deepen one's self-knowledge.

The first step was to assemble materials by displaying three different experiments related to the establishing of a relationship between pressure (weight) and volume. A second set of materials

⁹ See footnote 3, page 1.

¹⁰ Bernard Lonergan, *Insight: A Study in Human Understanding*, (1992, CWL 3, University of Toronto Press) Ch. 3, "The Canons of Empirical Method".

¹¹ The nature of the intelligibility of the universe implies that classical laws are always in play and the same at all times and places under similar conditions enabling for science to be possible. See A. D'Abro, *The Rise of the New Physics*, (1952, Dover Pub, Inc., NY) p. 179 on the expression of physical laws in differential equations.

¹² Peter Bergmann, *The Riddle of Gravitation*, (1968, Charles Scribner's Sons, NY) P. 140-143. Specifically, the Third Law of Forces: "To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

¹³ The experimenter controls the addition of weight or pressure by adding equal amounts at each time of measurement. The further question is what controls the decreasing amount of the gas so that its rate of decrease forms an intelligible ratio?

consisted of the various lists of measurements observed during the three experiments.¹⁴ A first insight revealed, as a result of what questions, that when weight or pressure is added, the volume of air decreases. A third set of materials consisted of the products of the two lists of measurements of weight or pressure and volume. A second insight into this data revealed that this third column of numbers were similar in approximate value.

There has been a shift of attention from outer circumstances in the visual experiments and the lists to inner circumstances when the various insights revealed the different intelligibilities immanent in the data. What are primary are the insights that revealed the intelligibilities. These insights are now our assembled materials. Note that the insights occur as a flow of intelligibilities.

Now, what does such a procedure offer for a science? These insights that occurred set up the possibility of working out correlations of correlations of correlations.¹⁵ In doing so, we get a glimpse of the unfolding of our own heuristic dynamic. That dynamic assists in the ordering of insights in an intelligible sequential series setting up the possibility of a genetic control of meaning by working out the relations between the different insights. Distinguishing between the different types of questioning and the different type of insights and working out the relations between the insights assists in understanding why the order of the emerging insights is as it is. This enables, for the individual, the possibility of providing an adequate history of a science's development, as well as what development is. Once this order is understood one can work out how progress occurs and what progress is.

Assembling the insights provides an entirely different process and procedure than merely attending to the data of sense. This assembling required a shift of attention. This shift from outer circumstances to inner operations orchestrates a move towards reversing the counterpositions of reductionism and positivism. It reveals that abstraction, through insight, is the procedure that adds the operations that lead to concepts and knowledge. This understanding of the insights also enables the teacher the ability to provide adequate pedagogy that does not violate the student's cognitional dynamic and discourages memory and rote learning which is often quite common in classrooms. A better understanding of Generalized Empirical Method¹⁶ is made possible for the individual who enters into the process of trying to understand relations in an experiment such as is provided above while being aware of the different operations and their content.

¹⁴ It is to be noted that we are not measuring pressure and volume in our first two experiments. We are taking readings from rulers and comparing those readings. Only in experiment 3 is pressure in psi, and volume in mls compared.

¹⁵ Terrance Quinn, *Generalized Empirical Method: In Philosophy and Science*, (2017, World Scientific Publishing Co. Pte. Ltd, Singapore) Pp. 21-26. Quinn offers a discussion on the triplicity of correlations using Galileo's experiments on falling bodies.

¹⁶ Bernard Lonergan, *A Third Collection*, (1985, Paulist Press, NY) P. 141. "Generalized empirical method operates on a combination of both the data of sense and the data of consciousness; it does not treat of objects without taking into account the corresponding operations of the subject; it does not treat of the subject's operations without taking into account the corresponding objects."

The procedure of working out the relationships involved in the experiments and various measurements requires shift in attention. It is to be noted that the insights that occur in this process reveal intelligibilities that a relationship between pressure (weight) and volume exists but these insights do not explain what that relationship is. Is it enough to say that because the experiments result in an approximate constant that an explanation is provided for that relationship? Something to think about?

There are two different types of insights relevant here, those revealing a relationship and those that would offer an explanatory account of what that relationship is.¹⁷ I have focused only on the first type of insight here. More work would be required to reach the second type of explanatory insights.¹⁸ It is this second type of insight that leads to the explanatory account of any datum and if accompanied by reflection on performance lead to adequate self-knowledge.

Earlier in this essay I stated that adequate self-appropriation cannot be achieved without some experience of science. Perhaps in the experience of this exercise one can achieve a glimpse of the meaning of that statement in noticing how different operations are called forth when involved in trying to understand a classical law. Such development with the addition of statistical method provides the possibility of communicating with scientists on their level increasing the probabilities of effectively reversing their counter positions. Without such development in myself I cannot talk adequately to a scientist about Generalized Empirical Method or Functional Specialization. Implementation is not possible.

It remains for further work to refine the explanatory process in terms of what is defined by Generalized Empirical Method. I am of the opinion that a full self-appropriation of the data of consciousness cannot be achieved without experience in scientific work¹⁹ that strives for an explanatory account of some data.²⁰ I welcome the experience of others on my opinion.

This exercise can assist one in understanding the existence and influence that our own counter positions still orchestrate over our own intellectual efforts. What counter positions are operating

¹⁷ This procedure would require work in the kinetic theory of gases and an understanding of molecular bonding and quantum theory.

¹⁸ The distinction between these two types of insights indicates a distinction between intellectual and theoretic conversion. See *Searching for Cultural Foundations*, ed. by Philip McShane (1984, University Press of America, Lanham, USA). P. vi-vii. McShane sets up that difference descriptively in the Preface. Intellectual conversion is an awareness of one's intellectual operations, and theoretic conversion is an allegiance to the explanatory or higher viewpoint that occurs only after much effort at attempting to understand such things as "electrons, or stones". This effort offsets the general bias of common sense as omniscient.

¹⁹ It brings to mind a question asked of Lonergan in the late 1970s at a Boston Workshop, "How much physics need a theologian know?" Lonergan replied: "Well, he should be able to read *Lindsay and Margenau*".

²⁰ I would also include self-attention in art or in historical writing as means to self-appropriation. What scientific self-appropriation brings to all these realms of meaning is a procedure for a refinement and a better control of meaning. See "A Concise Primer on Lonergan's Theory of Art: Part 1: Elemental Meaning and the Artist's Idea" by Patrick Brown and Michael Shute for an Introduction into self-appropriation into the aesthetic at Academic.edu. For related articles see The Journal of Macrodynamical Analysis, Volume 6 (2011) <https://journals.library.mun.ca/ojs/index.php/jmda/issue/view/22>

in present scientific procedures is a different question to be met with a different procedure. Only if one has worked in such a procedure as outlined here and in Lonergan's directions provided in *Insight's* first four chapters can one acknowledge the errors in one's own notion of objectivity and knowing. Some development in addressing our own counter positions is accomplished by working with these experiments in terms of appreciating the difficulty in understanding just what you are doing when measuring.²¹ This work should force a shift from intelligent observing to understanding. Reflection on the operations gradually moves one away from intelligent observing to organizing our intelligence. The position we slowly arrive at is a better appreciation of what abstraction is. The movement from outer circumstances to inner operations is a conscious one of shifting awareness. Once we become aware of our operations and the role they play in understanding we are challenging our own naïve realism (the already out there now real) and any naive position we may have on objectivity.

That appreciation also sheds some light on the heuristics of space time. How is this so? When we made the shift from observing the experiments to reflecting on numbered values and then through insight, grasped the intelligibility of the constant character of the product results, we were no longer observing the data of sense; we were arriving at intelligibilities beyond the original data of the experiments. We were moving towards the experience of abstraction. If we develop differential equations that express the generalization of Boyle's law we will have moved towards a fuller experience of abstraction as an invariant expression that is verifiable and not subject to the space or time that the experimental data are subject to.

As much as my experiments dealt with only spatial entities²² it perhaps provides some pointers towards understanding Lonergan's statement that Chapter Five of *Insight* forms a natural bridge from an examination of empirical science to an examination of common sense. "The abstract intelligibility of Space and Time...(is) of physical objects in their spatiotemporal relations." My

²¹ I had a similar experience when doing functional research in neuroscience in 2011. Published in *Seeding Global Collaboration* (2016, Axial Pub., Vancouver, BC) Chapter One: "Functional Research in Neuroscience. That research brought to light questions for me that I later addressed in 3 articles published now in *Global Collaboration: Neuroscience as Paradigmatic* (2016, Axial Publishing, Vancouver, BC). This research work revealed to me one of the functions of doing research. It raised questions for me that could only be addressed by the specialty Interpretation. I attempted to outline the source of the counter positions in neuroscience and offer accounts of what would help in reversing those counter positions. That entire bit of work took five years and perhaps provides a glimpse of my slow rate of growth in understanding but perhaps more importantly the difficulty in communicating with a science. If you do not have access to the book, the articles are published at the *Journal of Crossing Dialogues Association*, Rome, Italy at <http://www.crossingdialogues.com/journal.htm> 2013, Vol. 6, Issue 2, 2015, Vol. 8, Issue 1 and 2016, Vol. 9, Issue 1. The articles assembled materials in a descriptive manner. They are not science per se. Was my effort at communication adequate? Something for dialectic to ponder over in reaching for more adequate forms of communication.

²² Time would not be a variable in this experiment. It could be performed at day or night or winter or summer with no variance. Temporality would be an existential variable relating to my time that it took to arrive at some understanding of the experiments which is unrelated to the intelligibilities discovered. What would be variables are the temperatures of the liquids or gases used and the particular gases and liquids that are used in the experiment and the height above sea level which alters atmospheric pressure. But even if the variables are changed for subsequent experiments, the constancy of product values will be maintained for each individual experiment verifying that $P = V \times P$. What is necessary is that these variables do not change while one particular experiment is being carried out.

experiments were an effort to understand the spatial relations immanent in the process of measuring. For we must go beyond believing Lonergan to verifying his statement and this exercise seems to point to achieving that by personally participating in some scientific exercise. So I began with spatial marks on a ruler and moved to listing those spatial marks as numbers in a list. Due to the approximate constant of the products of the two lists I concluded that there is a relationship between the spatial numerical listings-correlations of correlations of correlations. That constancy reveals that an intelligibility of that particular spatial relation of those physical objects exists.

Now, a heuristic is defined as a method or procedure of doing something and our experiments are three different ways of performing the same experiment so qualify as heuristic generators of our whatting which comes alive with wanting to understand. I am trying to get my reader to their heuristic and that method or procedure pertains to the functioning of our intellectual operations. Any experiment, such as those provided in this paper, has the potential to offer experience of our own heuristic, if we are curious.²³

Conclusion

I began this essay with a quotation of Lonergan's regarding adequate self-knowledge. That is the objective of this exercise through the doing of a bit of science.²⁴ The nominal definition of science that is only procedural is that science is the relating of things to things. That definition does not provide one with an experience of science, of explanatory thinking or of one's heuristic. The experience of puzzling out some unknown can help us unravel our own counterpositions by assisting us in becoming more familiar with our own operations, our own heuristic dynamism.

As much as the sciences are mired in the counterpositions, it has been stated²⁵ that Lonergan scholarship suffers in a different way due to some form of **assumption** that they have personally moved beyond the counter positions²⁶ while avoiding science and probably Chapter Five of *Insight*, thereby having little empirical notion of what abstraction is or why Chapter Five forms a natural bridge from an examination of empirical science to an examination of common sense. Understanding the experiments manifests the difficulty in understanding within a scientific

²³ Robert Henman, *Reorienting Education and the Social Sciences: Transitioning towards the positive Anthropocene* (2019 Amazon.ca), chapters One and Two on curiosity. See also Philip McShane, *Futurology Express*, (2013 Axial Publishing, Vancouver, BC) Chapters 3 and 4 on whatting, the origins of our heuristic. I used McShane's book as a course text in Philosophy courses for 5 years in an effort to support my own emphasis in jump starting students' heuristic.

²⁴ The hope and heuristic is that in the long term Cosmopolis will be seeded, even if only as a small group.

²⁵ Philip McShane has made this statement often in his writings over the years. It would be an interesting research for some doctoral candidate to study that situation.

²⁶ In June of 1978 Phil McShane and I flew to my first Boston Conference. I was excited about going and meeting Lonergan. Phil replied; "Don't expect too much." I was two decades away from some understanding of what he meant and that gap is still challenging my own slow growth. I attended further conferences held in Boston until the late 1980s and continued to read articles in the Method Journal and various texts published in Lonergan scholarship while working in some science to try and appreciate this existential gap or even if there was a gap. It was more of a feeling at that time. In 2002 McShane held conferences in which he used "simple" experiments to engage participants in the process of scientific understanding. Only then did I realize my gap. In 2010 I began to do some study in physics which still continues today helping to close the gap.

context.²⁷ It provides a small increase in understanding the difficulty of our growth and a better appreciation for the need to grasp what Lonergan was up to.

This exercise reflects the meaning of Lonergan's first line in *Insight*.

*"In the midst of that vast and profound stirring of human minds, which we name the Renaissance, Descartes was convinced that too many people felt it beneath them to direct their efforts to apparently trifling problems."*²⁸

²⁷ It is to be noted that this exercise is not within the context of modern science. These exercises are within the context of the Classical Kinetic Theory of gases. Noticing the approximate constant of the products oscillating about one value is a glimpse of statistical theory in a rudimentary manner. Later work in the New Quantum Statistics would supersede the Classical Kinetic Theory of gases and provide a more accurate account of the relationship between pressure and volume. It is interesting to note that where in the first and second objectifications an example of correlations of correlations of correlations of the classical type was provided, the emerging context of quantum statistics reveals a correlation between classical and statistical theories that lays the foundations for a theory of Emergent Probability in light of the complementarity between the two theories.

²⁸ *Insight* (1992) 27.